of Chicago), a student of Yuri Manin in the 1970s, and Pierre Deligne (Princeton), who visited Manin in Moscow in 1972. "Manin was not in danger," says Deligne in the film, "but he suffered because his students suffered … their possibilities were very limited although they were very good." One of them was Michael Tsfasman, who had started his studies at Moscow State University in 1971. He reports in the film how two thirds of the students from Highschool No. 2, a top level college in Moscow, were not given the possibility to study because they were Jews, not communists or simply independent thinkers. Michael Tsfasman says he only got a chance to study at Moscow University because he was selected for the Russian team at the International Mathematical Olympiad (IMO); IMO participants were allowed to enter Russian top universities without exams and other selection processes.

Manin's life changed a lot after the fall of the Iron Curtain when he was allowed to travel again; for many years he had received invitations to congresses and meetings without being able to go to them. Friedrich Hirzebruch, professor of mathematics at Bonn University and the founder of the Max Planck Institute for Mathematics in Bonn, had invited Russian Mathematicians to Bonn every year since 1957, among them Yuri Manin. And Manin went to Bonn with four other mathematicians, on his way back to Russia from Paris in 1967. He gave a lecture about a problem in algebraic geometry at one of Hirzebruch's Arbeitstagungen. Hirzebruch reports in the film that he invited Manin again and again during the Cold War. After the Iron Curtain had finally fallen Manin accepted some invitations from Western universities and – enjoying the work abroad – resigned from his professorship at Moscow State University. One day Hirzebruch – close to his own retirement – visited Manin at Harvard and asked him to become his successor at the Max Planck Institute for Mathematics in Bonn; eventually Manin accepted. Another co-director of the Max Planck Institute of Mathematics, Don Zagier, talks in the film very warmly about Manin's style, about his greatest talent: how Manin was often able to see unexpected and surprising connections between two different branches of mathematics where no connections had appeared before – and establishing them successfully with creative ideas and persistency.

The last part of the movie is about Manin's childhood; it shows where and how he grew up. When Manin was five years old, he lost his father in World War II. His education started at School No. 7 in Simferopol, the capital of the Crimea; his first contact with another language was when he read Gulliver's Travels in English. Manin takes the viewers of the film back to his hometown Simferopol. We see Manin visiting the two houses of his childhood after decades of absence.

Then the title "Late Style" is explained at last. "Late style I think quite well explains this emotional atmosphere of returning to one's remote past," says Manin. "It's kind of you want to connect the beginning and the end of your life … you want to see some kind of entirety …"

Photo taken by Michael Ebner

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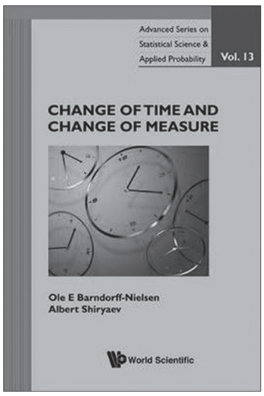

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Change of Time and Change of Measure

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Reviewer: Juan-Pablo Ortega

Mathematical finance is one of the most recent examples in the long list of successful instances of cross-fertilisation between pure mathematics and a domain of human knowledge in need of a rational and quantitative formulation of natural questions associated to it. Since Galilean times and up to the last century, this symbiosis linking mathematics to other disciplines took place mainly in the arenas of natural sciences and engineering. The important development during the last century of the mathematics of randomness (probability theory, theory of stochastic processes, statistical modelling, to give a few names) has created an array of powerful tools capable of handling the complex phenomena and uncertain outcomes usually studied within the realm of social sciences. Indeed, experience has shown that a probabilistic treatment of many problems arising in, for example, demography, economics, epidemiology and finance is much more pertinent than the one coming from classical deterministic methods. Even though this conceptual leap took some time to be assimilated, its result seems to be by now an established certitude.

This book takes two major ideas at the core of mathematical finance, namely the use of equivalent (martingale) measures and the equivalent (change of time and stochastic volatility) representations of a stochastic process, and uses them as a motivation to describe a variety of topics in stochastic analysis. The result is a beautiful and well-articulated monograph, full of information, where the interplay between deep mathematical ideas and extremely explicit and applied financial problems and their solutions is generously exemplified.

The change of time and the stochastic integral representation problems aim at rewriting a given stochastic process in terms of a simpler one (it can be a Brownian motion or, in general, a semimartingale) via a stochastic change of time and stochastic integration, respectively. The first approach is analysed in Chapter 1 of the book, in which all the necessary concepts having to do with stopping times and random changes of time are described, as well as the Dambis-Dubins-Schwarz Theorem that explains how any continuous martingale can be obtained out of a Brownian motion via a random change of time. The integral representation problem is tackled in Chapters 2 and 3 and gives the authors the opportunity to construct in a reduced number of pages a delightful presentation of the most relevant results in relation with stochastic integration and stochastic differential equations.

The existence and uniqueness results that are presented in those pages and that solve the described representation problems are not just deep and beautiful mathematical theorems but they also constitute key results in the understanding of important questions in mathematical finance. This interplay is presented mainly in Chapters 10 and 11. Indeed, the change of time representation provides mathematical legitimacy to a common interpretation of volatility among finance practitioners that consists of visualising moments of high fluctuation of the prices with an acceleration of the so-called operational or business time. Moreover, the existence of an integral representation for a process with a predictable integrand amounts in financial language to the availability of a self-financing trading strategy that replicates (or hedges, in financial jargon) a derivative product whose underlying asset has a dynamical behaviour described by the process in question. This makes of this mathematical construction, together with the notion of martingale or risk-neutral measure that we will review later on, a main building block of the no-arbitrage pricing and hedging theory of derivative securities. Indeed, when the underlying asset is described by a lognormal process (also called geometric Brownian motion), it gives rise to the Black-Scholes-Merton formulas for the price and the hedges of European style options, for which Robert Merton and Myron Scholes received in 1997 the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel (usually referred to as the Nobel Prize in Economics) and that made viable the massive and industrial scale trading of these securities.

Changes of measure are treated in Chapters 6 and 7. The goal is again reducing a given stochastic process to a simpler one by using in this case an equivalent probability measure; more specifically, the procedure aims at constructing a new measure that is absolutely continuous with respect to the original one, such that the law of the stochastic process under study "seen with the eyes" of this new measure coincides with the one coming from a simpler process like Brownian motion. There are several constructions available in the literature that serve this purpose. The book examines two of them: the Girsanov Theorem, of much use in mathematical finance, and the Esscher Transform, introduced initially in the context of actuarial sciences. Both procedures are constructive and provide new measures under which the stochastic process we are interested in becomes a martingale.

Again, the impact of these captivating mathematical constructions goes beyond a pure mathematical interest and has far-reaching implications in mathematical finance that are spelled out in Chapters 10 and 11. Indeed, the importance of the existence of an equivalent martingale measure for a (discounted) price process is captured in the so-called First Fundamental Theorem of Arbitrage Theory that establishes the equivalence between the availability of these measures and the absence of arbitrage opportunities in the market under study. This link is of paramount importance because many constructions in finance have the nonexistence of arbitrage opportunities, i.e. the impossibility of guaranteed profits without risk taking, as a fundamental hypothesis; this assumption is very plausible in highly liquid and efficient markets where results coming from mathematical finance are regularly applied.

Another fascinating link between two of the core ideas in the book, namely the existence of martingale measures and of integral representations and their application in mathematical finance, is provided by the Second Fundamental Theorem of Arbitrage Theory, also examined in Chapter 10. When we are in the presence of an underlying asset whose dynamics is such that there exists an integral representation for any contingent product, we say that the corresponding market is complete; according to what we said above, in complete markets any derivative product can be perfectly hedged/replicated. The Second Fundamental Theorem of Arbitrage Theory establishes the equivalence between the completeness of a market and the uniqueness of an equivalent martingale measure.

The book does not restrict itself to stating the basic concepts underlying mathematical finance and their connection with stochastic calculus that are available in so many other monographs. Indeed, it manages to present in a reduced number of pages most models that are used by practitioners when trying to solve the deficiencies of the Black-Scholes-Merton model that have been profusely documented over the years. Chapter 9 opens this discussion in the discrete time setup by introducing the ARMA/GARCH parametric family of time series models driven by a variety of different innovations (Inverse Gaussian, Generalised Inverse Gaussian, Generalised Hyperbolic, etc.) whose use aims at appropriately modelling stylised features of time series of financial returns that are empirically observed, like, for example, leptokurtosis (the likelihood of extreme events is higher than the one associated to the Gaussian distribution), volatility clustering (moments of high volatility tend to accumulate in time) or asymmetry (bad market days create more volatility than good ones). The analogue of this discussion in continuous time is carried out in Chapter 12, which contains a pleasantly readable presentation of stochastic volatility and Lévy processes introduced in the light of the integral and change of time representations, respectively, at the heart of the book.

Even though the book includes self-contained accounts of a variety of topics in stochastic analysis, like stochastic integration, stochastic differential equations, equivalent martingale measures, Lévy processes and stochastic volatility models, to name a few, it is not written as a textbook and is not well suited for a reader who would like to use it as an entry door to the field; the proofs for most results are not included, the book is not linear and, despite the presence of numerous examples, the book contains no exercises. Like most of the titles in World Scientific's Advanced Series on Statistical Science and Applied Probability, the reader is expected to have some acquaintance with the various subjects covered by the volume. The style of presentation and the contents makes this enjoyable book ideal for a reader interested in going deeper in understanding the area or in having a panoramic view of it. For that category of readers, this book will certainly provide an opportunity to learn additional results and to establish beautiful connections between them, all of it elegantly illustrated with one of the most relevant and impressive applications of this mathematical field.

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